ULTIMATE STRENGTH THEORY OF RAFT SLABS ON MOISTURE REACTIVE SOIL

1. Notation & units

B_w = Stiffening rib width (m)
h = Width of soil-slab contact area (m)
k = Soil stiffness (kPa/m)
L = Width of equivalent rectangular floor plan (m)
h = Soil-slab contact width
L_s = Isolated slab panel width (m)
M_u = Overall stiffened section ultimate bending moment per metre (kNm/m)
m_u = Isolated slab panel ultimate bending moment per metre (kNm/m)
P_s = Total foundation load (kN)
p_0 = Initial state soil pressure
p^* = Peak ultimate soil pressure (kPa)
W_e = Edge load per unit perimeter length (kN/m)
W_s = Total weight of superstructure (kN)
w_f = Uniformly distributed floor load (kPa) (Dead plus live)
y_s = Surface movement (mm)
α = Aspect ratio of equivalent rectangular floor plan (≥1.0)
α_s = Aspect ratio of isolated slab panel (≥1.0)
γ = Ratio $\frac{h}{L}$
Δ = Deflection (mm)

2. Equivalent rectangular floor plan

Most residential floor plans consist of overlapping rectangles; therefore analysis of an equivalent rectangular plate model is a good starting point. Figure 1 shows a typical residential floor plan and equivalent rectangle. The equivalent rectangle is calculated as follows:

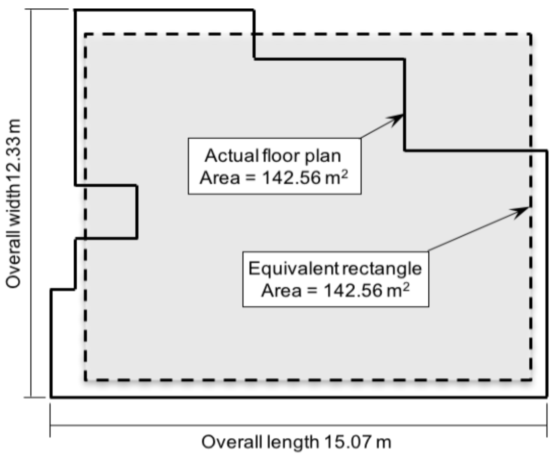

Figure 1 - Typical residential floor plan and equivalent rectangle

The dimensions of the equivalent rectangle are calculated as follows:

$$\text{Aspect ratio} = \frac{15.07}{12.33} = 1.222 \qquad \text{Width} = \sqrt{\frac{142.56}{1.222}} = 10.8\ \text{m} \qquad \text{Length} = 1.222 \times 10.8 = 13.2\ \text{m}$$

3. Theory

3.1 Limit state methodology

Limit state methodology relies on the worst-case scenario of identifying and analyzing the governing failure mechanism and applying appropriate factors of safety and design criteria to ensure adequate in-service performance. The major performance parameters are deflection, bending moment, and soil pressure. In the context of residential slabs on moisture reactive soil, these performance parameters are inextricably linked through soil-structure interaction.

The three self-evident conditions for failure of a slab are:

- Yield condition
- Equilibrium condition
- Mechanism condition

The yield line method satisfies these conditions perfectly.

3.2 Global subsidence failure

3.2.1 Yield line pattern

Figure 2 shows the fully developed yield line pattern for global subsidence failure of a rectangular floor slab. The shaded area represents the soil-slab contact area. Assuming surface movement outside the rectangular soil-slab domain is constant all around the perimeter, the soil-slab contact area is also rectangular, with edges equidistant from the slab edges. Hence the corner yield lines roughly pass through the corners of the soil-slab contact rectangle.

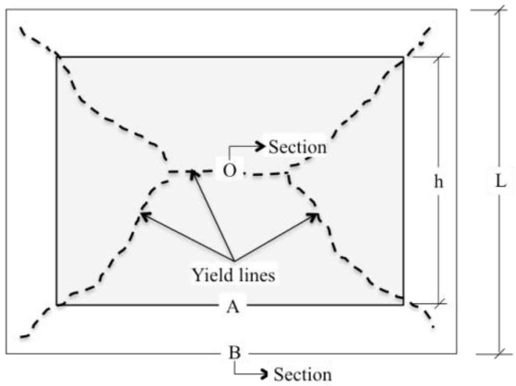

Figure 2 – Fully developed yield line pattern & soil-slab contact area

The yield line pattern for global subsidence failure develops as follows: The first yield line more than likely starts along the center line parallel to the long side of the equivalent rectangle, where the bending moment is greatest. As surface movement increases, the central yield line lengthens and then splits into two yield lines heading at about 45^0 angles to the corners. When the yield lines are fully developed, distortion of the slab is like the motion of a three-dimensional mechanism consisting of four plane segments hinged together along the yield lines and rotating relative to each other about the yield lines. In subsiding soil conditions, the distorted slab is domed, and in heaving soil conditions it is dished.

3.2.2 Ultimate pressure distribution on soil-slab interface

In common with most interactive soil-structure analyses it is assumed that soil is a linearly elastic material. It follows from previous paragraphs that the ultimate pressure on the soil-slab interface varies linearly from zero at point A on the edge of the soil-slab contact rectangle to a peak at point O at the centre of the slab, as shown in Figure 3.

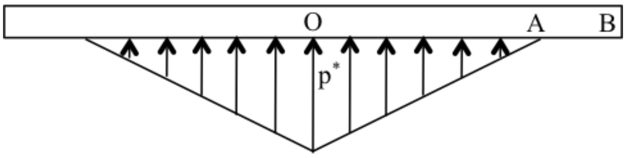

Figure 3 - Ultimate subsidence pressure distribution on soil-slab interface

3.2.3 Ultimate deflection analysis

Figure 4 shows the half-section through the fully developed yield line pattern.

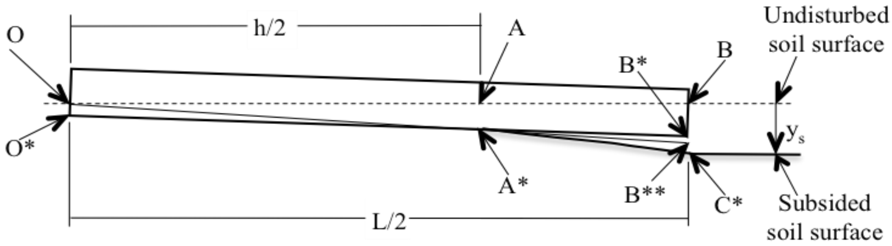

Figure 4 - Half-section through fully developed yield line pattern

Points O, A, and B are in a straight line along the undisturbed soil surface. Points O*, A*and B* are on the under-side of the slab. Point C* is on the displaced soil surface at the edge of the non-contact zone. Point B** is on the extension of the straight line through points O and A* to the edge of the slab. The soil surface profile in the non-contact zone is indeterminate.

The soil-slab interface pressure at O* is p* and therefore, by definition of soil stiffness, $\frac{p^*}{k}$ is the amount the centre of the slab is depressed into the soil. The interface pressure at A* is zero and therefore $\frac{p^*}{k}$ is also the relative deflection of the interface between O* and A*. The relative deflection across the full width of the slab is determined by scaling along the straight line O*A* to B* : $D = \frac{p^*}{gk}$.

Surface movement is determined by scaling along the straight line OA* to B**, which includes a reasonable allowance for the indeterminate gap between the underside of the slab and soil surface: $y_s = \frac{2p^*}{gk}$

Interestingly, the ultimate relative deflection equals exactly half the surface movement.

3.2.4 Equilibrium analysis

Peak pressure is derived by equating the total foundation load on the slab to the upward soil pressure resultant on the soil-slab contact rectangle. Ultimate bending moment is derived from conventional yield line analysis. These derivations involve integration of products of areas of rectangles and triangles. The end results are:

$$p^* = \frac{6P_s}{g(3a + 2g - 3)L^2} \qquad M_u^+ = \frac{6W_s + [(3a + 1)w_f - g^2(a + g - 1)p^*]L^2}{24(a + 1)}$$

3.3 Global heave failure

The yield line pattern for global heave failure is identical to that for hogging failure, except as follows: The yield lines are negative bending moment, the soil-slab contact area is the unshaded area in Figure 2, and the peak soil-slab interface pressure is at the slab edges. The end results are:

$$p^* = \frac{6P_s}{g(3a - 2g + 3)L^2} \qquad M_u^- = \frac{6W_s + (3a+1)w_f L^2 - g[3(a+1) - g(a - g + 3)]p^* L^2}{24(a+1)}$$

3.4 Local slab panel failure

The failure mechanism for isolated internal slab panels is similar to global hogging failure. The critical slab panel is at the centre of the floor plan, where the soil-slab interface pressure is greatest. Yield line analysis includes negative bending moment yield lines on the panel edges in addition to positive bending moment yield lines in the interior. The result is: $m_u^+ + m_u^- = \frac{(3a_s - 1)p^* L_s^2}{48(a_s + 1)}$

3.5 Local corner and edge failures

Figure 5 shows the yield patterns for local corner and edge failure mechanisms. These types of failures may occur for example on a cut/fill site where the fill has not been adequately compacted around a corner or along an edge, or on a site with gilgais. The following results are obtained from straightforward equilibrium analysis:

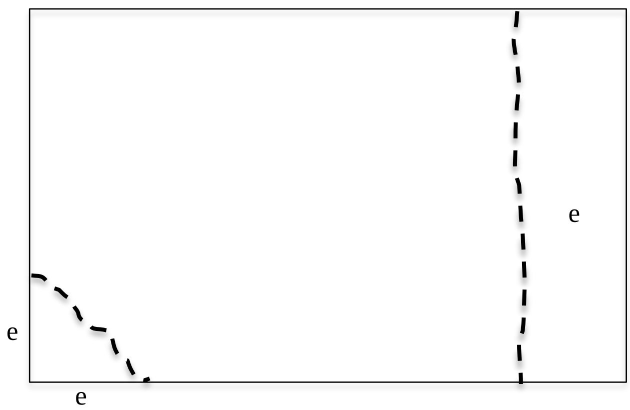

Figure 5 - Yield lines for local corner and edge failures

Local corner failure: $M_u = \frac{W_e e}{2} + \frac{w_f e^2}{12}$ Local edge failure: $M_u = W_e e(1 + \frac{e}{L}) + \frac{w_f e^2}{2}$

The problem with these failure mechanisms is that there is no way of relating them to surface movement. Designers wishing to explore corner and edge failures might consider taking distance e in Figure 5 as the edge distance defined in AS-2870.

3.6 Governing failure mechanism

"Like the strength of a chain is that of its weakest link…the strength of a slab is that of its weakest failure mechanism"

According to the upper/lower bound theorems in structural mechanics the governing failure mechanism for slabs on reactive soil is the one exhibiting the lowest ultimate surface movement capacity. Conversely, the governing failure mechanism is the one requiring the highest ultimate strength to sustain the most severe combination of design loading and surface movement.

It is impossible to predict a-priori which failure mechanism governs. Global subsidence failure usually governs on building sites under normal soil moisture conditions at time of construction. Global heave failure may govern on building sites that are abnormally dry at time of construction, followed by prolonged wet weather. Local corner and edge failures may govern on building sites exhibiting non-uniform soil characteristics, such as for example inadequately compacted soil on a cut-fill site, or the presence of gilgais. Local slab panel failure may govern when stiffening ribs are too widely spaced or when the slab thickness is too small, or when it supports excessive local point loads.

3.7 Ultimate strength performance

The ultimate strength performance of slabs on moisture reactive soil is a complex function of interactive slab and soil properties, static loading, and dynamic surface movement. A typical graph of ultimate bending moment versus surface movement at which the slab fails is shown in Figure 6. This characteristic "*S-curve*" graph consists of a straight line for a short distance on either side of the vertical axis, followed by exponential curves to subsidence and heave asymptotes.

The easiest way to plot this graph is to make γ in the equations in Section 3 the independent variable. Calculate the values of all the other variables in the equations, starting with γ equal to 1.00 and decrementing it by small amounts. Continue until the calculated y_s values cover the desired range between negative and positive values. This procedure produces the exponential parts of the graph, and a straight line drawn between the points γ equal to 1.00 on either side of the vertical axis completes the graph.

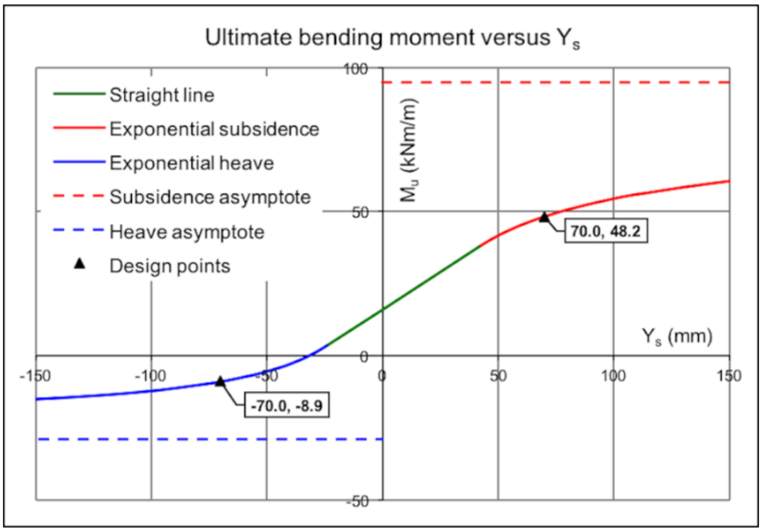

Figure 6 – Characteristic ultimate strength performance graph & design chart

The straight-line part of this graph is in the range of surface movements: $\frac{-12P_s}{(3a+1)kL^2} £ y_s £ \frac{12P_s}{(3a-1)kL^2}$

The gradient of the straight line portion of the bending moment graph is: $\frac{(3a^2-1)kL^2}{288a(a+1)}$

The asymptotic limits are: Subsidence: $M_u^+ = \frac{6W_s + (3a+1)w_f L^2}{24(a+1)}$ Heave: $M_u^- = \frac{-(3a-1)w_f L^2}{24(a+1)}$

3.8 Ultimate strength design

Design ultimate bending moments are determined by interpolating the performance graph in Figure 6 at specific and appropriately factored Y_S values applicable to the building site. Designers can then work out the slab dimensions, stiffening rib layout, steel reinforcement and other details in accordance with conventional structural design practice, including compliance with Australian Standards AS 2870 and AS 3600 and other relevant documents. It can be demonstrated that graphs of ultimate deflection versus Y_S are always straight lines with a gradient equal to exactly 0.5, enabling designers to assess the potential damage category of slab designs in accordance with AS 2870 Table C2. Whilst the necessary mathematical process for the recommended ultimate strength design methodology is quite tedious, the number-crunching can be easily programmed in a spread sheet for routine application in practice.